

Restoring genuine tripartite entanglement under local amplitude damping

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Abstract. We investigate the possibility to restore genuine tripartite entanglement under local amplitude damping. We show that it is possible to protect genuine entanglement using CNOT and Hadamard gates. We analyze several ordering of such recovery operations. We find that for recovery operations applied after exposing qubits to decoherence, there is no enhancement in lifetime of genuine entanglement. Actual retrieval of entanglement is only possible when reversal scheme is applied before and after the decoherence process. We find that retrieval of entanglement for mixture of $|\widetilde{W}\rangle$ state with white noise is more evident than the respective mixture of $|W\rangle$ state. We also find the retrieval of entanglement for similar mixture of $|GHZ\rangle$ state as well.

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1. Introduction

Entanglement among more than two particles is one of the peculiar features in quantum physics and its characterization is an active area of studies [1,2]. Pure state entanglement is useful for certain tasks like teleportation or cryptography, but in reality unavoidable interactions with environment leads to degradation and even abrupt elimination of entanglement from quantum states [3]. Therefore it is important to study the techniques to protect entanglement from decoherence. One specific type of decoherence is amplitude damping, which is mainly present in ion trap experiments, like atomic qubits subjected to spontaneous emission. Recently, three main strategies were proposed to combat amplitude damping. First technique is called weak measurement reversal [4], in which environment is monitored to restore entanglement probabilistically. Second technique is called quantum measurement reversal [5], where a partial measurement maps a qubit towards ground state before amplitude damping and later another measurement restores the initial state. Third scheme is to use Hadamard and CNOT gates to restore a single qubit pure state [6], an arbitrary two qubit pure state [7], and mixed states of two qubits in a weak measurement [8].

Effects of decoherence on entanglement among multiparticle states have extensively been studied and this is an active area of research [9–16]. Several works considered the

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life time of entanglement under decoherence [9,10]. Here, the life time of entanglement denotes the time with a nonzero value of the chosen measure of entanglement. Some works studied bipartite aspects of the entanglement of several particles [11], however it gives a partial description, because multiparticle entanglement is different from entanglement between all bipartitions [2]. Another main problem behind previous studies is lack of availability of a fully developed theory of multiparticle entanglement. Hence, one could only make statements about lower bounds on entanglement instead of its actual value [12]. The exact calculation of a multiparticle entanglement measure was possible for special states and decoherence models [13]. Recent progress in the theory of multiparticle entanglement, especially the computable entanglement monotone for genuine multiparticle entanglement from Ref. [17], enabled us to study the effects of decoherence on multiparticle entanglement [16].

In our current study, we utilize the computable genuine negativity [18] to analyze the possibility to restore genuine entanglement between three qubits undergoing local amplitude damping. We investigate the effects of applying local recovery operations, which are composed of Hadamard and CNOT gates, in several ordering to find the optimum results. We find that optimum restorage of genuine entanglement is only possible for the case, when the recovery operations are applied both before and after the decoherence process. In all other cases, the life time and amount of genuine entanglement is not restored considerably. We analyze the mixtures of W-type states and GHZ-type states with white noise (maximally mixed state). We find that locally equivalent state to W-state is more restored in comparison with W-state, because the error matrix has zero entries for off-diagonal elements for $|\widetilde{W}\rangle$ state. Although we talk about multipartite quantum systems and multiparticle entanglement, however we study three two-level quantum systems (or three qubits). As the discussion on genuine entanglement is general and applicable for an arbitrary finite dimensional quantum system (quNit) and finite number of parties (M), this means that results are applicable to multi-qubit systems.

This work is organized as follows. In Section 2.1, we describe amplitude damping model to obtain the dynamics of an arbitrary density matrix. In Section 2.2, we briefly review the concept of multiparticle entanglement and also review the derivation of genuine negativity. In Section 2.3, we define the investigated recovery schemes. We present the main results in Section 3. Finally, we offer some conclusions in Section 4.

2. Preliminaries

2.1. Local amplitude damping model for multiparticle states

We consider N qubits (e.g., N two level atoms) which are coupled to their own local reservoirs. The reservoirs are assumed to be independent from each other. We assume weak coupling between each qubit and the corresponding reservoir and no back action effect of the qubits on the reservoirs. We also assume that the correlation time between the qubits and the reservoirs is much shorter than the characteristic time of the evolution

so that the Markovian approximation is valid. The interactions of the physical system with environment is usually studied either by solving a master equation, using the Kraus operator formalism, and quantum trajectories. We work in the Kraus operator formalism. The time evolution of an initial density matrix can be written as

$$\varrho(t) = \sum_{i=1}^{2^N} K_i(t) \varrho(0) K_i^\dagger(t), \quad (1)$$

where $K_i(t)$ are the Kraus operators, satisfying the normalization condition $\sum_i K_i^\dagger(t) K_i(t) = \mathbb{1}$ and N is the number of qubits. The precise form of these Kraus operators are given as $K_i(t) = \omega_{i_1}^A \otimes \omega_{i_2}^B \otimes \cdots \otimes \omega_{i_N}^N$, where ω_i^j are the single-qubit Kraus operators acting on the j th qubit.

For *amplitude damping*, there are two Kraus operators for a single qubit,

$$\omega_1^j = \begin{pmatrix} 1 & 0 \\ 0 & \gamma_j \end{pmatrix}, \quad \omega_2^j = \begin{pmatrix} 0 & \sqrt{1 - \gamma_j^2} \\ 0 & 0 \end{pmatrix}, \quad (2)$$

where $\gamma_j = e^{-\Gamma_j t/2}$ and Γ_j is the spontaneous emission decay rate of j th qubit. In ion-trap experiments, amplitude damping is the typical noise. For the case of 3-qubit states, there are 2^3 global Kraus operators $K_i(t)$ for the amplitude damping channels. For the sake of simplicity we assume onwards that $\gamma_A = \gamma_B = \cdots = \gamma_N = \gamma$.

The time evolved density matrix for a single qubit can directly be computed. Under amplitude damping it is given as

$$\varrho(t) = \begin{pmatrix} \varrho_{11} + \varrho_{22}(1 - e^{-\gamma t}) & \varrho_{12} e^{-\gamma t/2} \\ \varrho_{21} e^{-\gamma t/2} & \varrho_{22} e^{-\gamma t} \end{pmatrix}, \quad (3)$$

where ϱ_{ij} are initial density matrix elements. For more qubits, the calculation of density matrices is straightforward.

2.2. Genuine multiparticle entanglement and genuine negativity

We review the basic definitions of genuine entanglement and genuine negativity by taking three parties A , B , and C , with generalization to more parties as straightforward. A state is separable with respect to some bipartition, say, $A|BC$, if it is a mixture of product states with respect to this partition, that is, $\rho = \sum_j p_j |\psi_A^j\rangle\langle\psi_A^j| \otimes |\psi_{BC}^j\rangle\langle\psi_{BC}^j|$, with p_j a probability distribution. We name these states as $\rho_{A|BC}^{sep}$. Similarly, we can write other states as $\rho_{B|CA}^{sep}$ and $\rho_{C|AB}^{sep}$. A state is biseparable if it is convex combination of these states, that is

$$\rho^{bs} = q_1 \rho_{A|BC}^{sep} + q_2 \rho_{B|CA}^{sep} + q_3 \rho_{C|AB}^{sep}, \quad (4)$$

with $\sum_i q_i = 1$. Finally, a state is genuinely entangled if not biseparable. In this paper, we always mean genuine multipartite entanglement when we talk about entanglement.

Meantime a technique has been developed to detect and characterize multipartite entanglement [17]. The method is based on using positive partial transpose (PPT) mixtures. A bipartite state $\rho = \sum_{ijkl} \rho_{ij,kl} |i\rangle\langle j| \otimes |k\rangle\langle l|$ is PPT if its partially transposed

matrix $\rho^{TA} = \sum_{ijkl} \rho_{ji,kl} |i\rangle\langle j| \otimes |k\rangle\langle l|$ has no negative eigenvalues. The separable states are always PPT [19]. The set of separable states with respect to some partition is therefore contained in a larger set of states which has a positive partial transpose for that bipartition. The PPT states with respect to some bipartition are $\rho_{A|BC}^{PPT}$, $\rho_{B|CA}^{PPT}$, and $\rho_{C|AB}^{PPT}$. If a state can be written as

$$\rho^{PPTmix} = r_1 \rho_{A|BC}^{PPT} + r_2 \rho_{B|CA}^{PPT} + r_3 \rho_{C|AB}^{PPT}, \quad (5)$$

it is called a PPT mixture. Here $\sum_i r_i = 1$. Any biseparable state is a PPT mixture, hence any state which is not a PPT mixture is guaranteed to be genuinely entangled. The major advantage of taking PPT mixtures instead of biseparable states is the fact that PPT mixtures can be fully characterized by the method of semidefinite programming (SDP), a standard method in convex optimization [20]. Generally the set of PPT mixtures is a very good approximation to the set of biseparable states and delivers the best known separability criterion for many cases; however, there are genuine entangled states which are PPT mixtures [17].

It was proved [17] that a state is a PPT mixture iff the optimization problem

$$\min \text{Tr}(\mathcal{W}\rho) \quad (6)$$

with constraints that for all bipartition $M|\bar{M}$

$$\mathcal{W} = P_M + Q_M^{T_M}, \quad \text{with } 0 \leq P_M \leq 1 \text{ and } 0 \leq Q_M \leq 1 \quad (7)$$

has a positive solution. Here T_M means partial transpose with respect to partition M . The constraints means that the operator \mathcal{W} is a decomposable entanglement witness for any bipartition. For a negative minimum, ρ is not a PPT mixture and hence is genuinely entangled. For the semidefinite program (SDP), the minimum can be efficiently computed and the optimality of the solution can be certified [20]. For solving the SDP we used the programs YALMIP and SDPT3 [21], an implementation which is freely available [22].

This approach can be used to quantify genuine entanglement and the absolute value of this minimization was proved to be an entanglement monotone for genuine entanglement [17, 18]. We denote this measure as genuine negativity $E(\rho)$ or E -monotone [18]. For bipartite systems, this monotone is equivalent to *negativity* [23]. For a system of qubits, this measure is bounded by $E(\rho) \leq 1/2$ [17].

2.3. Recovery schemes

The basic recovery scheme, which we utilize to study our problem consist of Hadamard and CNOT gates. It was shown that an arbitrary single qubit state can be completely recovered using this technique [6]. Later on, this method was generalized to restore two qubits quantum states and entanglement [7, 8]. In our work, we analyze several scenarios where these local operations are performed on an arbitrary quantum state of three qubits. We extend the previous studies in two ways. First, we study this scheme for multipartite quantum systems and second we study the various ordering of applying these recovery operations and their effect on restorage of genuine entanglement.

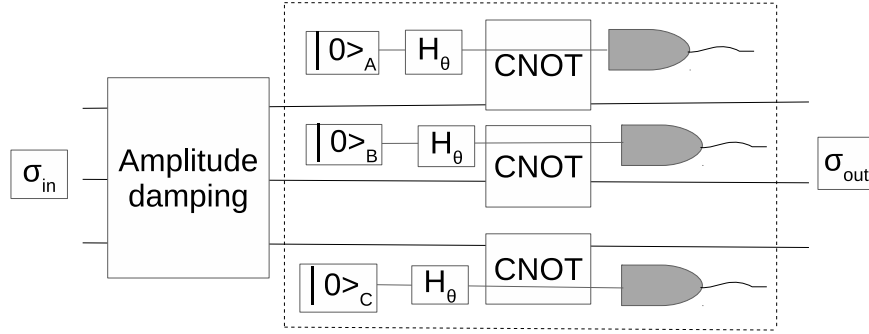


Figure 1. Circuit diagram for recovery operations to restore genuine entanglement for three-qubit states under amplitude damping. See text for further details.

The recovery schemes can be outlined as follows. First, we take an arbitrary initial quantum state σ_{in} . In addition to initial state, we take three auxiliary qubits which are in $|000\rangle$ state. To create superpositions of ground and excited state, a Hadamard gate is applied on each auxiliary qubit individually, such that

$$\rho_{Ai} = H_{\theta}(|0\rangle\langle 0|)H_{\theta}^{\dagger} = \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{pmatrix}, \quad (8)$$

where

$$H_{\theta} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}. \quad (9)$$

After this step, three CNOT gates are separately applied to each pair of system qubit in state σ_{in} and auxiliary qubit with system qubit as control qubit and auxiliary qubit as target qubit. After this procedure, we take measurements on auxiliary qubits. If the measurement results are $|000\rangle$, then the recovery process is successful otherwise fails. The probability of this success process can be worked out easily, which depends on when we choose to apply these operations. The dashed box in Figure (1) depicts the essential recovery operations. The final state after taking measurements on auxiliary qubits is denoted by σ_{out} .

First scheme, we focus on the case when recovery operations are applied only for one time and that after the initial state σ_{in} is exposed to local amplitude damping. This situation is shown in the circuit diagram of Figure (1). We will show the effects of this scheme on genuine entanglement in next section. Here we just want to mention that similar to case of two qubits [7, 8], here these operations are unable to affect the dynamics of genuine entanglement in a considerable way. Only the numerical value of entanglement is slightly enhanced. This means that it is not possible to delay the life time of entanglement with these operations and genuine negativity vanishes at the same time as it vanishes without any recovery operations. For this scheme, we choose $\tan \theta = 1/\gamma$ to compensate for decoherence effects.

Another question, we can ask is that what happens if we apply two consecutive recovery operations after the qubits are exposed to amplitude damping channels.

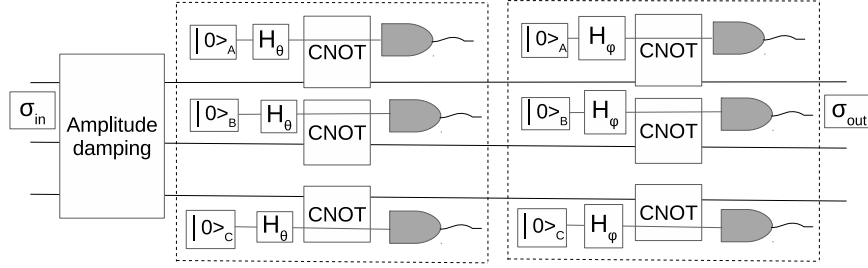


Figure 2. Circuit diagram for two consecutive operations to restore genuine entanglement for three-qubit states under amplitude damping. See text for further details.

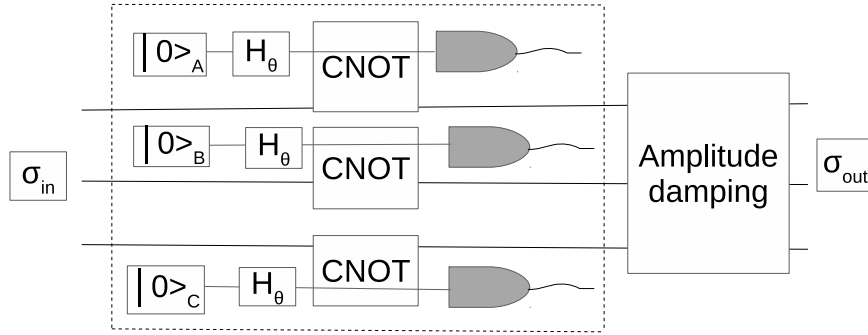


Figure 3. Circuit diagram for single recovery operations applied before the three-qubit states are exposed to amplitude damping. See text for further details.

Figure (2) depicts this scheme. We will present the detailed results in the next section but an interesting feature is the observation that if we choose the relation “ $\tan \theta \tan \phi \gamma = 1$ ”, then applying recovery operations is equivalent to applying it for single time. Here θ (ϕ) denotes rotation angle in Hadamard gate for first (second) recovery operations. It means any number of such operations satisfying this condition do not affect the dynamics at all. Whereas if we choose $\tan \phi = 1/\gamma$ and keep $s = \tan \theta$, then still it is not possible to delay life time of entanglement. We discuss the detailed results in next section.

We have seen that by the above mentioned order of applying recovery operations, we do not get our aim, that is to enhance the life time of genuine entanglement against amplitude damping. Therefore, we apply recovery operations for single time, in order to make the initial state robust against amplitude damping. After this process, we expose the qubits to decoherence. The circuit diagram is shown in Figure (3). We will see in next section that the life time of genuine entanglement is enhanced in this case upto some extent.

Finally we consider the situation where we apply the recovery operations before and after the qubits undergo amplitude damping. We will see in next section that only in this case the entanglement is protected considerably. So this scheme is very effective in recovering genuine entanglement against amplitude damping. Figure (4) depicts the circuit diagram for this case. In this case, we have chosen the parameters such that

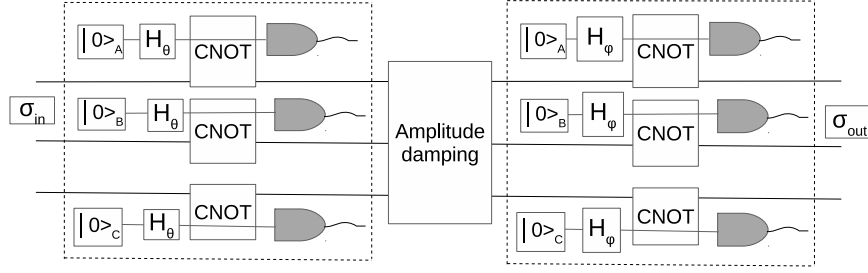


Figure 4. Circuit diagram for double recovery operations applied before the three-qubit states are exposed to amplitude damping and once afterwards. See text for further details.

$\tan \phi = 1/(x\gamma)$, where “ x ” denotes the parameter for first recovery operations.

We have observed that for schemes shown in Figure (1) and Figure (4), it is always possible to write the final density matrix ρ_f as sum of an initial state and an error matrix, that is,

$$\rho_f = \frac{1}{\mathcal{N}} [\rho_i + \rho_{err}], \quad (10)$$

where \mathcal{N} is the normalization factor and ρ_{err} is the error matrix given as

$$\rho_{err} = \begin{pmatrix} \tilde{\rho}_{11} & \tilde{\rho}_{12} & \tilde{\rho}_{13} & \tilde{\rho}_{14} & \tilde{\rho}_{15} & \tilde{\rho}_{16} & \tilde{\rho}_{17} & 0 \\ \tilde{\rho}_{21} & \tilde{\rho}_{22} & \tilde{\rho}_{23} & \tilde{\rho}_{24} & \tilde{\rho}_{25} & \tilde{\rho}_{26} & 0 & 0 \\ \tilde{\rho}_{31} & \tilde{\rho}_{32} & \tilde{\rho}_{33} & \tilde{\rho}_{34} & \tilde{\rho}_{35} & 0 & \tilde{\rho}_{37} & 0 \\ \tilde{\rho}_{41} & \tilde{\rho}_{42} & \tilde{\rho}_{43} & \tilde{\rho}_{44} & 0 & 0 & 0 & 0 \\ \tilde{\rho}_{51} & \tilde{\rho}_{52} & \tilde{\rho}_{53} & 0 & \tilde{\rho}_{55} & \tilde{\rho}_{56} & \tilde{\rho}_{57} & 0 \\ \tilde{\rho}_{61} & \tilde{\rho}_{62} & 0 & 0 & \tilde{\rho}_{65} & \tilde{\rho}_{66} & 0 & 0 \\ \tilde{\rho}_{71} & 0 & \tilde{\rho}_{73} & 0 & \tilde{\rho}_{75} & 0 & \tilde{\rho}_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (11)$$

where $\tilde{\rho}_{ij}$ are error matrix entries. The error matrix in both situations have exactly same zero matrix elements. The nonzero matrix elements depends on the type of recovery scheme and have different expressions for each recovery scheme. ρ_i is an arbitrary initial state of three qubits. It is interesting to note that for locally equivalent state to W -state given as

$$|\widetilde{W}\rangle = \frac{1}{\sqrt{3}} (|110\rangle + |101\rangle + |011\rangle), \quad (12)$$

has zero entries in error matrix, that is, $\tilde{\rho}_{46} = \tilde{\rho}_{47} = \dots = \tilde{\rho}_{76} = 0$. This means that for an initial $|\widetilde{W}\rangle$ state, the error matrix would have minimum entries, and better recovery of quantum state and entanglement.

In rest of this work, by ρ_d , we mean density matrix under decoherence without any type of recovery operations. By ρ_r , we mean the recovery scheme applied only once and that after exposing qubits to amplitude damping (Figure (1)). By ρ_{dr} , we mean double recovery scheme in which recovery operations are applied before and after the qubits are exposed to local amplitude damping (Figure (4)).

3. Results

It is well known that for three qubits, there exist two inequivalent genuine multiparticle entangled states, which can not be transformed into each other by any SLOCC [2], namely the *GHZ* states and the *W* states,

$$\begin{aligned} |GHZ\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), \\ |W\rangle &= \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle). \end{aligned} \quad (13)$$

For the *GHZ* state, genuine negativity has a value of $E(|GHZ\rangle\langle GHZ|) = 1/2$, while for the *W* state, its value is $E(|W\rangle\langle W|) \approx 0.443$. We consider the effects of applying recovery schemes on mixture of *GHZ* state with maximally mixed state, given as

$$\rho_{GHZ} = \alpha |GHZ\rangle\langle GHZ| + \frac{1-\alpha}{8} I_8, \quad (14)$$

where $0 \leq \alpha \leq 1$ and I_8 is 8×8 identity matrix. The entanglement properties of these states are well known and entanglement criterion discussed in previous section is also necessary and sufficient for detection of genuine entanglement of such mixtures [17]. Similarly, we can write the equation for mixture of *W* state as

$$\rho_W = \alpha |W\rangle\langle W| + \frac{1-\alpha}{8} I_8, \quad (15)$$

We set $\alpha = 0.8$ for these class of states in this work to compare the behaviour of recovery operations.

As discussed earlier, if qubits are first exposed to amplitude damping and after that we apply recovery operations for a single time then there is no effect on the life time of genuine entanglement except that its numerical value is slightly enhanced. On the other hand, if we apply the recovery operations two times after decoherence (see Figure (2)), then the effects on dynamics depends on $\tan \theta$ (rotation angle for first recovery operation) and $\tan \phi$ (rotation angle for second recovery operation). For the choice that meet the condition “ $\tan \theta \tan \phi \gamma = 1$ ”, we find that dynamics is not affected at all, and second operations acts as identity matrix. However, if we set “ $s = \tan \theta$ ”, and choose “ $\tan \phi \gamma = 1$ ”, then the dynamics is quite different for each value of “ s ”. Figure (5) shows genuine negativity plotted against the parameter Γt for various values of “ s ”. We observe that for lower values of “ s ”, the amount of initial entanglement also becomes lower, however, interestingly each curve comes to an end at a same point $\Gamma t \approx 0.46$, irrespective of initial amount of genuine entanglement.

Figure (6) shows similar results for an initial mixture of *W* states. Once again we see the same trend as in the case of *GHZ* states except that decay is relatively faster. For all choices of parameter “ s ”, genuine entanglement ends at $\Gamma t \approx 0.84$. So we conclude that all such schemes in which recovery operations are performed after the qubits are exposed to amplitude damping can not enhance the life time of genuine entanglement.

Let us now focus on the second category of recovery scheme, where we apply recovery operations before qubits are allowed to interact with the environment. This

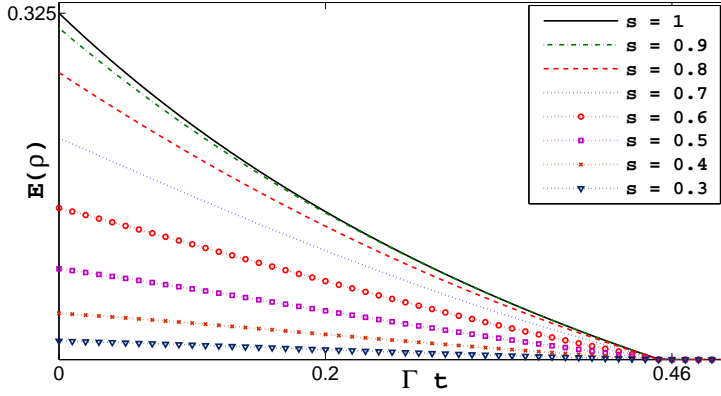


Figure 5. Entanglement monotone plotted against parameter Γt for states Eq.(14) with $\alpha = 0.8$ and various values of s .

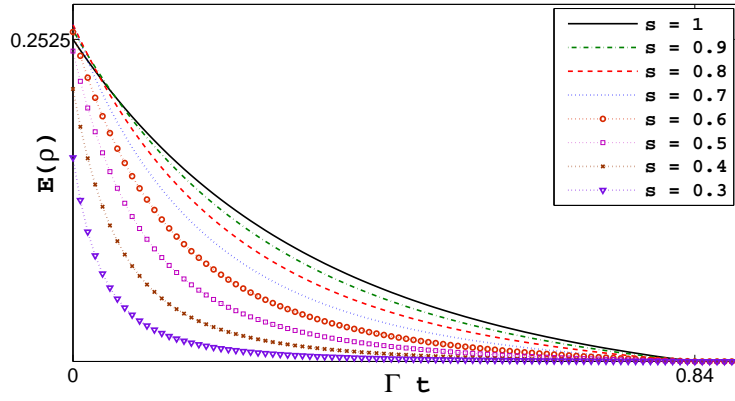


Figure 6. Entanglement monotone plotted against parameter Γt for states Eq.(15) with $\alpha = 0.8$ and various values of s .

scheme can be further divided into two cases. In the first case, we apply recovery operations only initially and then let the qubits undergo decoherence process. In second case, we apply recovery operations before and after the qubits are exposed to amplitude damping noise (see Figures (3) and (4)). We find out that only in this category, the life time of genuine entanglement can be enhanced considerably.

Figure (7) depicts the results for *GHZ* mixture for various values of parameter “ $x = \tan \theta$ ”. We conclude two things from this figure. First, decreasing the value of “ x ” prolongs the life time of genuine entanglement and second the actual initial amount of genuine entanglement also decreases by decreasing the parameter “ x ”. However, we should bear in mind that decreasing parameter “ x ” means the decreasing probability for the successful recovery of state and protocol. However, we see that this scheme can delay the vanishing of genuine entanglement with some compromise on failure of the recovery protocol.

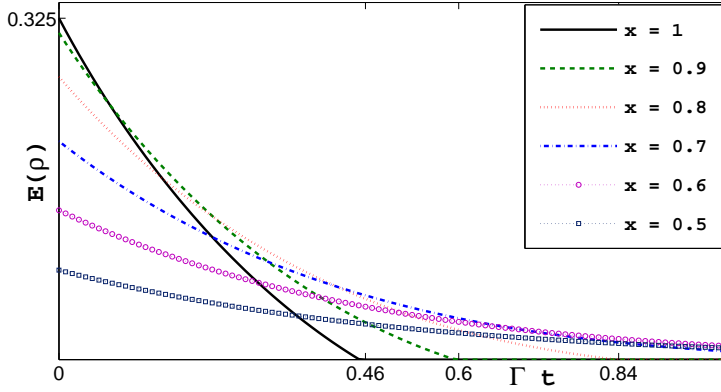


Figure 7. Genuine negativity for mixture of three-qubit GHZ states is plotted against parameter Γt and various values of parameter “ x ”.

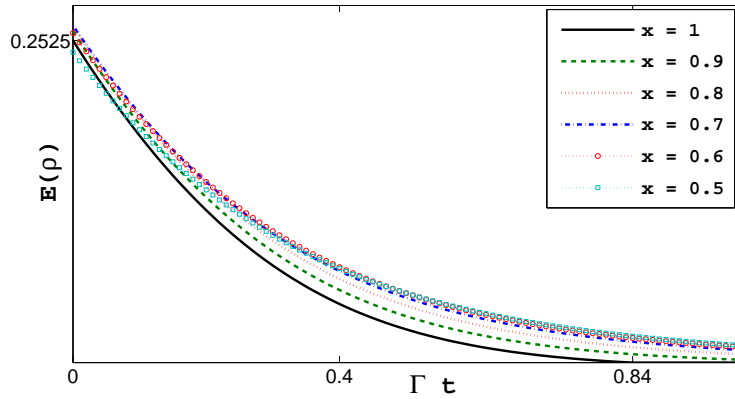


Figure 8. Genuine negativity for mixture of three-qubit W states is plotted against parameter Γt and various values of parameter “ x ”.

Figure (8) shows the results for similar mixture of W state. We see similar trend in the prolongation of life time of genuine entanglement, however as compared with mixtures of GHZ state, the amount of initial entanglement do not vary much. Actually for some values of parameter “ x ”, the initial entanglement increases slightly as well.

Finally, we consider the case where we apply recovery operations for two times. Once before the qubits interact with environment and once after that. This case we name as double recovery of genuine entanglement, denoted with a “dr” in subscript of density matrix in the figures and text. Figure (9) show the effects of double recovery for mixture of GHZ state. The solid (black) line is for amplitude damping without any recovery operations. The dashed (blue) line is for recovery operations applied for single time and after that qubits are exposed to amplitude damping. We see that these two lines almost overlap each other and both end at $\Gamma t \approx 0.46$. The other three curves are for decreasing values of parameter “ x ” and we see that as “ x ” decreases the recovery of genuine entanglement is considerably enhanced. Another interesting feature is that the

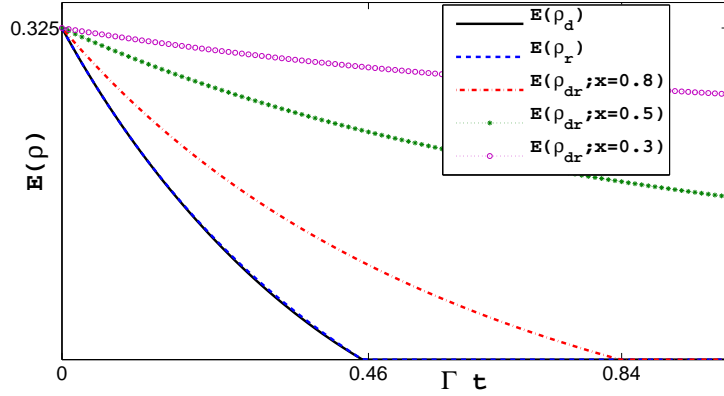


Figure 9. Genuine negativity for mixture of three-qubit GHZ state plotted against parameter Γt . The three curves are for double recovery operations.

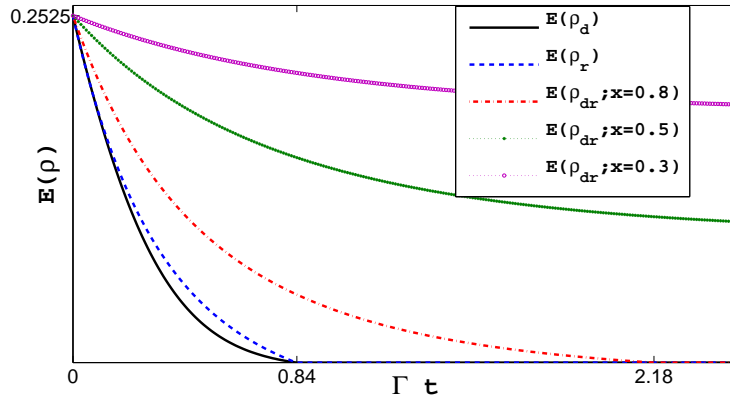


Figure 10. Genuine negativity for mixture of three-qubit W state against parameter Γt . The three curves for double recovery operations reflects the recovery of genuine entanglement.

initial amount of genuine entanglement remains same for all values of “ x ”.

Figure (10) shows the effects of double recovery operations for mixture of W state. The solid line is for dynamics without any recovery operations and dashed (blue) line is for operations applied before amplitude damping. Similar to mixture of GHZ state, both curves reach to zero at $\Gamma t \approx 0.84$, however, here numerical value of genuine entanglement is enhanced and can be seen more clearly. The other three curves show that recovery of genuine entanglement is greatly enhanced by decreasing the parameter “ x ”.

Finally we check the behaviour for mixture of $|\widetilde{W}\rangle$ state (Eq. (12)). Figure (11) depicts the effects for amplitude damping without recovery operations (solid line), recovery operations only at the start (dashed blue line), and double recovery operations (other three curves). Interestingly, in this case, the single recovery operations enhance the degree of genuine entanglement more clearly than W state. Also the other three curves shows that $|\widetilde{W}\rangle$ state is more robust against amplitude damping. This observation

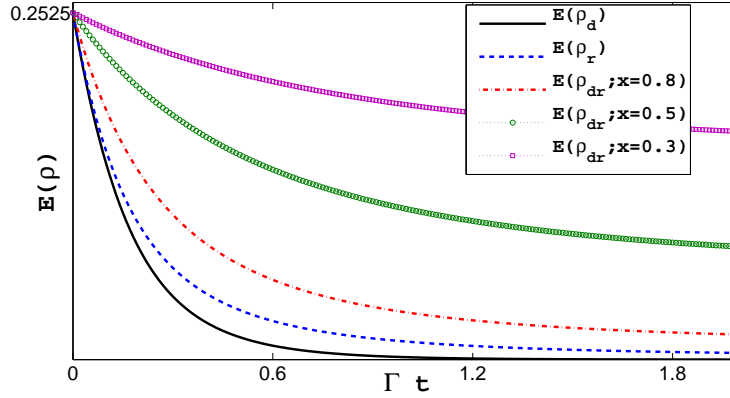


Figure 11. Genuine negativity for mixture of three-qubit $|\widetilde{W}\rangle$ state against parameter Γt . The recovery of genuine entanglement is more clearly enhanced for this initial state.

is interesting and surprising, as it has all three components as doubly excited as compared with $|W\rangle$ state. This state, we expected to be more fragile in the sense that it would lose its genuine entanglement more quickly, however, we have seen that it is exhibiting quite opposite behaviour. The reason for this behaviour can be understood by examining the error matrix Eq. (11), in which we already discussed that for $|\widetilde{W}\rangle$ state, the error matrix entries are zero, meaning better recovery.

4. Discussion and Summary

We studied the effects of local recovery operations on genuine multiparticle entanglement for three qubit quantum states undergoing local amplitude damping. We analyzed several schemes in which such recovery operation could be applied. We found that for all recovery operations applied after the qubits are exposed to amplitude damping noise, there is no enhancement in the lifetime of genuine entanglement. We have studied the mixtures of W-type states and GHZ state with white noise as examples. We found that for all such schemes in which the recovery operations are applied before, it is possible to prolong the lifetime of genuine entanglement up to some extent. For considerably and fully recovery of quantum states and genuine entanglement, the recovery operations must also need to be applied after the qubits are exposed to local amplitude damping. Only such double recovery scheme actually makes the quantum states robust against local amplitude damping. Our studies can be extended in several directions. First, it would be interesting to investigate the relation of time of applying second recovery operations with the decoherence time. In this work, we only consider the action of applying second operations at the start of the dynamics. It would be worth to figure out whether there is any critical time for applying such operations and after this time, it may not be possible to restore genuine entanglement. Second, it would be interesting to find out the similar recovery schemes for other types of decoherence models, like phase damping and/or depolarizing noise.

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